J-O-Caml (2)

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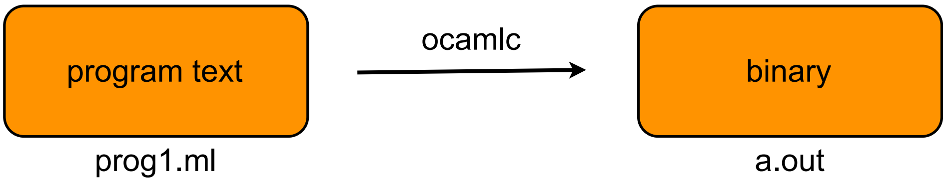
Plan of this class

- compiling programs
- list processing
- pattern-matching
- new data types
- labeling algorithm

Compiling programs



- `ocamlc -o prog1` produces `prog1` (executable byte code)
- `ocamlc -c prog1.ml` produces `prog1.cmo` (byte code)
- `ocamlopt -c prog1.ml` produces `prog1.cmx` (binary)
- also files `prog1.cmi` (compiled interfaces -- see later)

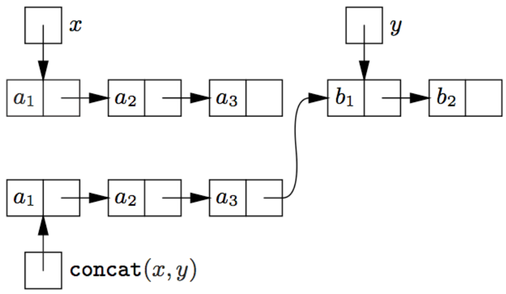
List processing

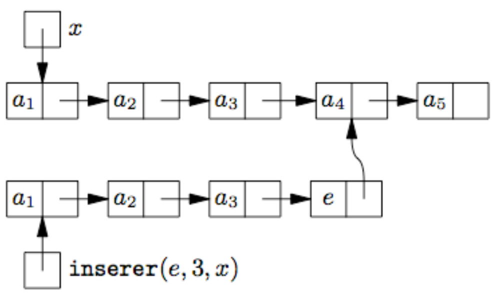
```
# let rec length x = match x with
  | [ ] -> 0
  | a :: x' -> 1 + length x' ;;

# let rec concat x y = match x with
  | [ ] -> y
  | a :: x' -> a :: concat x' y ;;

# let rec reverse x = match x with
  | [ ] -> [ ]
  | a :: x' -> concat (reverse x') [a] ;;

# let rec insert a n x =
    if n = 0 then a :: x else match x with
    | [ ] -> [ ]
    | b :: x' -> b :: insert a (n-1) x' ;;
```





List processing

```
# let rec length x = match x with
  | [ ] -> 0
  | a :: x' -> 1 + length x' ;;
    val length : 'a list -> int = <fun>
# let rec concat x y = match x with
  | [ ] -> y
  | a :: x' -> a :: concat x' y ;;
    val concat : 'a list -> 'a list -> 'a list = <fun>
# let rec reverse x = match x with
  | [ ] -> [ ]
  | a :: x' -> concat (reverse x') [a] ;;
    val reverse : 'a list -> 'a list = <fun>
# let rec insert a n x =
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      val insert : 'a -> int -> 'a list -> 'a list = <fun>
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List processing

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# let rec insert a n x =
    if n = 0 then a :: x else match x with
    | [ ] -> [ ]
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      val insert : 'a -> int -> 'a list -> 'a list = <fun>
```


- polymorphic types
- `'a list` (say ``alpha list") is type of list of any type, e.g. `int list`, `bool list`, `...`

Exercices on lists

Exercices on lists

- Arbitrary precision numbers can be implemented by lists (little endian or big endian style). Write addition and multiplication algorithms.
- Conway sequence of lists starts with list `[1]`. Then next list in sequence is obtainte by reading previous list. Therefore `[1; 1]`, `[2; 1]`, `[1; 2; 1; 1]`, `[1; 1; 1; 2; 2; 1]`.... Print lists of Conway sequence. Is there any unlucky number ?

New data types

- products

```
# let move (x, y) (dx, dy) = (x +. dx, y +. dy);;

# move (2.1, 3.0) (0.5, 0.5);;

# let dotProduct (x, y) (x', y') = (x *. x' +. y *. y');;

# let crossProduct (x, y) (x', y') = (x *. y' -. x *. y');;
```

New data types

- products

```
# let move (x, y) (dx, dy) = (x +. dx, y +. dy);;
val move : float * float -> float * float -> float * float = <fun>
# move (2.1, 3.0) (0.5, 0.5);;
- : float * float = (2.6, 3.5)
# let dotProduct (x, y) (x', y') = (x *. x' +. y *. y');;
val dotProduct : float * float -> float * float -> float = <fun>
# let crossProduct (x, y) (x', y') = (x *. y' -. x *. y');;
val crossProduct : float * 'a -> 'b * float -> float = <fun>
```

New data types

- sums

```
# type complex = Cartesian of float * float | Polar of float * float ;;

# let polar_of_cartesian c = match c with
  | Cartesian (x, y) -> Polar (sqrt(x *. x +. y *. y), atan (y /. x))
  | Polar (rho, theta) -> c ;;

# let cartesian_of_polar c = match c with
  | Polar (rho, theta) -> Cartesian (rho *. (cos theta), rho *. (sin theta))
  | _ -> c ;;

# let add c c' =
    let c1 = cartesian_of_polar c and c1' = cartesian_of_polar c' in
    match c1, c1' with
      | Cartesian(x, y), Cartesian(x', y') -> Cartesian(x +. x', y +. y')
      | _ -> failwith "Impossible" ;;
```

New data types

- sums

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# type complex = Cartesian of float * float | Polar of float * float ;;
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# let polar_of_cartesian c = match c with
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  | Polar (rho, theta) -> c ;;
    val polar_of_cartesian : complex -> complex = <fun>
# let cartesian_of_polar c = match c with
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    match c1, c1' with
      | Cartesian(x, y), Cartesian(x', y') -> Cartesian(x +. x', y +. y')
      | _ -> failwith "Impossible" ;;
        val add : complex -> complex -> complex = <fun>
```

New data types

- recursive types

```
# type tree = Empty of unit | Node of tree * int * tree;;

# let rec size a = match a with
  | Empty() -> 0
  | Node (left, _, right) -> 1 + size left + size right ;;

# size (Node(Empty(), 3, Node (Empty(), 4, Empty())));;
```

New data types

- recursive types

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# let rec size a = match a with
  | Empty() -> 0
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    val size : tree -> int = <fun>
# size (Node(Empty(), 3, Node (Empty(), 4, Empty())));;
- : int = 2
```

New data types

- recursive types

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# size (Node(Empty(), 3, Node (Empty(), 4, Empty())));;
```

- recursive polymorphic types

```
# type 'a tree = Empty of unit | Node of 'a tree * 'a * 'a tree ;;

# let rec size a = match a with
  | Empty() -> 0
  | Node (left, _, right) -> 1 + size left + size right ;;

# let a =  Node(Empty(), 3, Node (Empty(), 4, Empty()));;

# let b =  Node(Empty(), "nihao", Node (Empty(), "bushi", Empty()));;


# size b;;
- : int = 2
# size a;;
- : int = 2
```

New data types

- recursive types

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# type tree = Empty of unit | Node of tree * int * tree;;
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# let rec size a = match a with
  | Empty() -> 0
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- recursive polymorphic types

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# type 'a tree = Empty of unit | Node of 'a tree * 'a * 'a tree ;;
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# let rec size a = match a with
  | Empty() -> 0
  | Node (left, _, right) -> 1 + size left + size right ;;
    val size : 'a tree -> int = <fun>
# let a =  Node(Empty(), 3, Node (Empty(), 4, Empty()));;
val a : int tree = Node (Empty (), 3, Node (Empty (), 4, Empty ()))
# let b =  Node(Empty(), "nihao", Node (Empty(), "bushi", Empty()));;
val b : string tree =
  Node (Empty (), "nihao", Node (Empty (), "bushi", Empty ()))
# size b;;
- : int = 2
# size a;;
- : int = 2
```

New data types

- recursive polymorphic types (alternative definition for binary trees)

```
# type 'a tree = Leaf of 'a | Node of 'a tree * 'a * 'a tree ;;

# let rec flatten a = match a with
  | Leaf (x) -> [ x ]
  | Node (tleft, _ , tright) -> List.append (flatten tleft) (flatten tright) ;;

# let a = Node ( Leaf (3), 5, Node (Leaf (4), 7, Leaf (6))) ;;

# flatten a ;;

# let rec flatten a = match a with
  | Leaf (x) -> [ x ]
  | Node (tleft, _ , tright) -> (flatten tleft) @ (flatten tright) ;;
```

New data types

- recursive polymorphic types (alternative definition for binary trees)

```
# type 'a tree = Leaf of 'a | Node of 'a tree * 'a * 'a tree ;;
type 'a tree = Leaf of 'a | Node of 'a tree * 'a * 'a tree
# let rec flatten a = match a with
  | Leaf (x) -> [ x ]
  | Node (tleft, _ , tright) -> List.append (flatten tleft) (flatten tright) ;;
    val flatten : 'a tree -> 'a list = <fun>
# let a = Node ( Leaf (3), 5, Node (Leaf (4), 7, Leaf (6))) ;;
val a : int tree = Node (Leaf 3, 5, Node (Leaf 4, 7, Leaf 6))
# flatten a ;;
- : int list = [3; 4; 6]
# let rec flatten a = match a with
  | Leaf (x) -> [ x ]
  | Node (tleft, _ , tright) -> (flatten tleft) @ (flatten tright) ;;
    val flatten : 'a tree -> 'a list = <fun>
# flatten a;;
- : int list = [3; 4; 6]
```

Caring about space

- with an extra argument as an accumulator of the result

```
# let flatten a =
    let rec flatten1 a res = match a with
      | Leaf (x) -> x :: res
      | Node (tleft, _, tright) -> flatten1 tleft (flatten1 tright res)
    in flatten1 a [ ] ;;
        val flatten : 'a tree -> 'a list = <fun>
# flatten a;;
- : int list = [3; 4; 6]
```

Caring about space

- with an extra argument as an accumulator of the result

```
# let flatten a =
    let rec flatten1 a res = match a with
      | Leaf (x) -> x :: res
      | Node (tleft, _, tright) -> flatten1 tleft (flatten1 tright res)
    in flatten1 a [ ] ;;
        val flatten : 'a tree -> 'a list = <fun>
# flatten a;;
- : int list = [3; 4; 6]
```

Combien d'objets dans une image?

Jean-Jacques Lévy
INRIA

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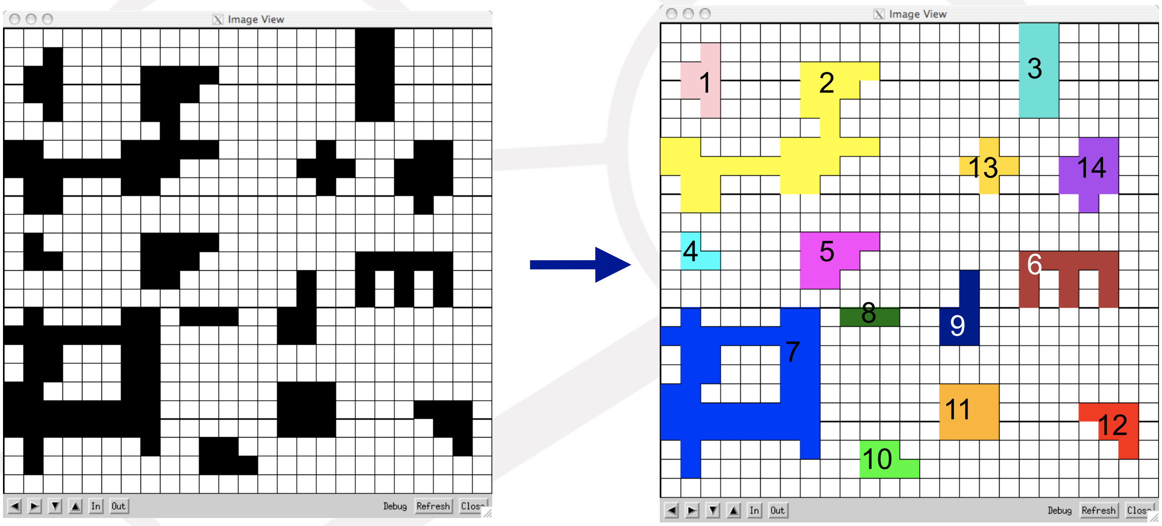
Labeling



16 objects in this picture

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Naive algorithm

1) choose an unvisited pixel
2) traverse all similar connected pixels
3) and restart until all pixel are visited

Very high complexity:

- how to find an unvisited pixel ?
- how organizing exploration of connected pixels (which direction?)

Algorithm

1) first pass

- scan pixels left-to-right, top-to-bottom giving a new object id each time a new object is met

2) second pass

- generate equivalences between ids due to new adjacent relations met during scan of pixels.

3) third pass

- compute the number of equivalence classes

Complexity:

- scan twice full image (linear cost)
- try to efficiently manage equivalence classes (Union-Find by Tarjan)

Animation

with Polka system for animated algorithms

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Exercise for next class

- find an algorithm for the labeling algorithm

Equivalence classes

Equivalence classes

"Union-Find"

- objects $x_1, x_2, \ldots, x_n$
- equivalence*s*
- find the equivalence class of x_p
- 3 operations:
  - $\mathtt{NEW}(x)$ new object
  - $\mathtt{FIND}(x)$ find canonical representative
  - $\mathtt{UNION}(x, y)$ merge 2 equivalence classes

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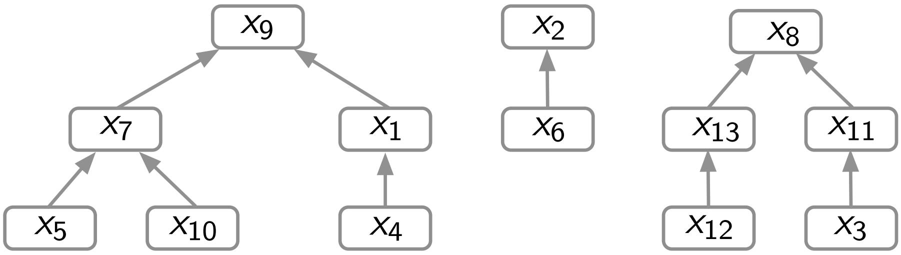
Equivalence classes

"Union-Find"

- with tree-like structure



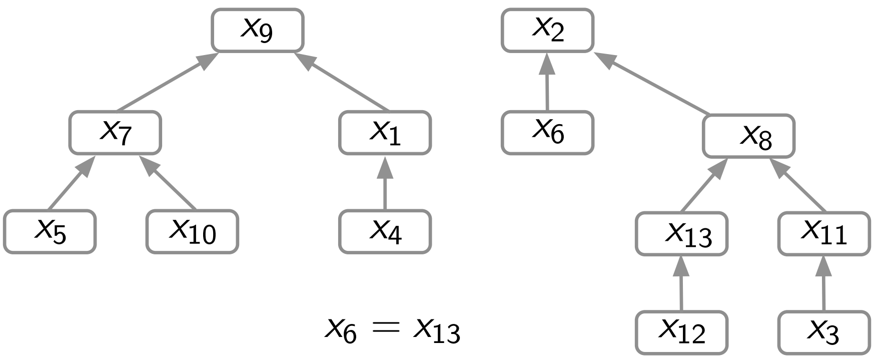
- given by the array of direct ancestors

`ancestor`	0	9	2	11	1	7	2	9	8	9	7	8	13	8

Equivalence classes

"Union-Find"

- unbalanced trees because of merges
  - $\mathtt{UNION}(x, y)$ merge 2 equivalence classes

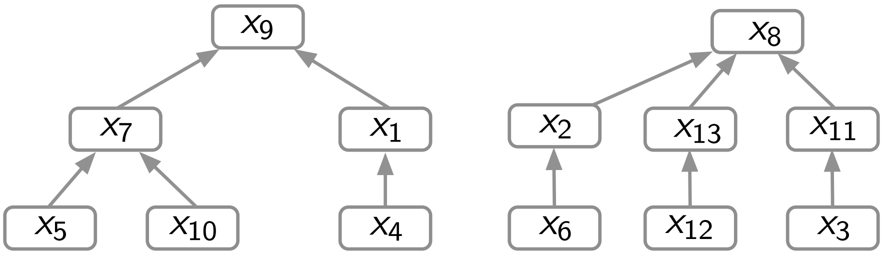


$x_6 = x_{13}$

Equivalence classes

"Union-Find"

- try to balance
  - $\texttt{UNION}(x, y)$ merge 2 equivalence classes



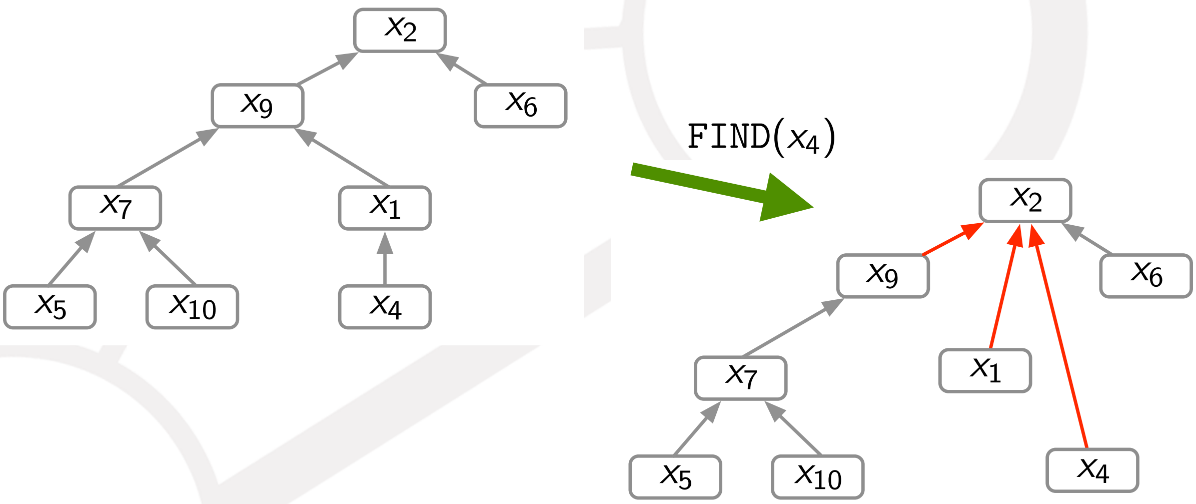
therefore keep height at each node

$O(\log n)$ opérations pour $\texttt{FIND}(x)$

Equivalence classes

"Union-Find"

- compression of path toward canonical representative
  - $\texttt{FIND}(x)$ with side-effect on tree structure



Exercice for next class

- find good primitives for graphics in Ocaml library
- design the overall structure of the labeling program