

# Formal Verification of a Concurrent Bounded Queue in a Weak Memory Model

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# Introduction

**contribution:**

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## tool:

Cosmo, our program logic for Multicore OCaml

# Sequential queues

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## A specification for sequential queues

$\{ \text{True} \}$	$\{ \text{IsQueue } q [v_0, \dots, v_{n-1}] \}$
make ()	enqueue $q$ $v$
$\{ \lambda q. \text{IsQueue } q [] \}$	$\{ \lambda (). \text{IsQueue } q [v_0, \dots, v_{n-1}, v] \}$
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behaviors of the program are interleavings of its threads

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can we keep the sequential spec? valid, but...

IsQueue  $q [v_0, \dots, v_{n-1}]$  is **exclusive**

$\implies$  effectively no concurrent usage

# Invariants

[in a concurrent separation logic such as Iris]

an **invariant** holds at all times

idea: the user shares  $q$  in an invariant:

$$I \triangleq \exists n, v_0, \dots, v_{n-1}. \text{IsQueue } q [v_0, \dots, v_{n-1}]$$

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[in Iris]

**logically atomic triples** are triples  $\langle \cdot \rangle \cdot \langle \cdot \rangle$  such that:

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$x$  binds things which are known only during that step

## A specification for concurrent queues under SC

$$\left\{ \begin{array}{l} \text{True} \\ \text{make } () \end{array} \right\} \quad \left\langle \begin{array}{l} n, v_0, \dots, v_{n-1}. \text{IsQueue } q [v_0, \dots, v_{n-1}] \\ \text{enqueue } q v \end{array} \right\rangle$$
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# Concurrent queues in weak memory

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# Weak memory models

## weak memory models:

each thread has its own **view** of the state of the shared memory

- example: C11
- example: Multicore OCaml

[Dolan et al, PLDI 2018, *Bounding data races in space and time*]

operational semantics with thread-local views

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operational semantics with thread-local views

**Cosmo**: a program logic for M-OCaml based on this semantics

[ICFP 2020]

based on Iris (hence: separation logic, ghost state, invariants)

assertions can be **subjective**: depend on current (thread's) view

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to be specified:  $\text{IsQueue } q [v_0, \dots, v_{n-1}]$  is objective

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let enqueue q =
```

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  let x = array[2] in
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  x[1] ← 3;
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```
  { x[1] ↗ 3 }
```

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  enqueue q x
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let $x = \text{array}[2]$ in	let $x = \text{dequeue } q$ in
$x[1] \leftarrow 3;$	{ $x[1] \rightsquigarrow 3$ }
{ $x[1] \rightsquigarrow 3$ }	do_something $x[1]$
enqueue $q$ $x$	

$x[1] \rightsquigarrow 3$  is **subjective**

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to be specified: dequeueer observes all writes done by enqueueer  
( $\implies$  “release-acquire” pattern)

a lattice of views (larger = more up-to-date)

## Views in Cosmo

a lattice of views (larger = more up-to-date)

new assertions:

$\uparrow \mathcal{V}$  “the ambient view contains  $\mathcal{V}$ ”  $\implies$  subjective

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## Transferring views through the queue

idea: pretend the queue stores the views being transferred

$$\text{IsQueue } q [ v_0 \quad , \dots, v_{n-1} \quad ]$$

the enqueuer pushes its view alongside the enqueued value:

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our spec is weaker (no guaranteed sync. from dequeuer to enqueueer)

⇒ covers more lock-free queues

## Conclusion

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- proof of a simple client
- machine-checked (Coq, Iris) 

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[a refinement proof in SC: Vindum & Birkedal, 2021, *Mechanized Verification of a Fine-Grained Concurrent Queue from Facebook's Folly Library*]

- proof of a simple client
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