Control structures, first lecture

The birth of control structures: from “goto” to structured programming

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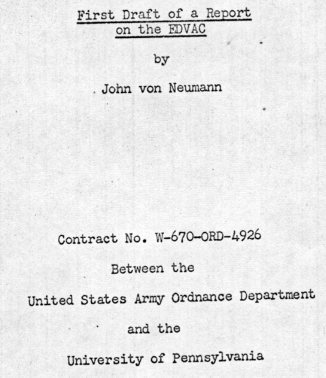
Early computers, early languages

The programmable computer with stored program

An architecture described in 1945 by John von Neumann and semi-independently by Alan Turing.

Used by most computers designed since then.



First Draft of a Report
on the EDVAC

by

John von Neumann

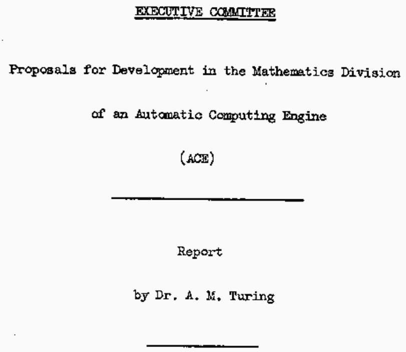
Contract No. W-670-ORD-4926

Between the

United States Army Ordnance Department

and the

University of Pennsylvania



EXECUTIVE COMMITTEE

Proposals for Development in the Mathematics Division

of an Automatic Computing Engine

(ACE)

Report

by Dr. A. M. Turing

Program counter and jumps

A “PC” register, the program counter, contains the address of the memory word containing the the next instruction.

Regular instructions increment the PC
$\rightarrow$ sequential execution.

Branch instructions set the value of the PC
$\rightarrow$ jump to any program point.

Example: output the numbers 1!, 2!, …, n!, …

```
 0: set r1, 1            (r1 = the value of n!)
 4: set r2, 1            (r2 = the value of n)
 8: output r1
12: add r2, r2, 1
16: mul r1, r1, r2
20: branch 8
```

Conditional branches

Modern processors provide conditional branch instructions that

- set the PC if a condition is true
  (e.g. if a register contains a non-zero value);
- continue in sequence if the condition is false.

Example: a counted loop that computes $n!$

```
                                (r1 = the value of n)
 0: set r2, 1
 4: mul r2, r2, r1
 8: sub r1, r1, 1
12: brnz r1, 4
                                (r2 = the value of n!)
```

Historical alternative: self-modifying code

Neither von Neumann nor Turing consider conditional branches. Instead, they rely on the code being stored in memory and modifiable during the execution of the program!

Example: branch to 0 if $\texttt{r1} = 0$ and to 4 if $\texttt{r1} = 1$.
We assume that the `branch` *n* instruction is encoded as $\texttt{0x76000000} + n$.

```
184: set r2, 0x76000000
188: mul r3, r1, 4
192: add r2, r2, r3
196: store r2, 200
200: nop
```

(dynamically-modified instruction)

An encoding of instructions and their operands as binary numbers (often 32-bit wide).

Example: conditional branch in the Power architecture.

bc | Base | User | **bc**

Branch Conditional [and Link] [Absolute]

bc	BO,BI,BD	(AA=0, LK=0)
bca	BO,BI,BD	(AA=1, LK=0)
bcl	BO,BI,BD	(AA=0, LK=1)
bcla	BO,BI,BD	(AA=1, LK=1)

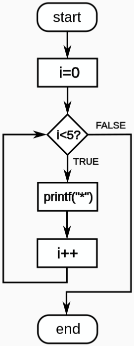
0–5	6–10	11–15	16–29	30	31
0 1 0 0 0 0	BO	BI	BD	AA	LK

```
if ¬BO_2 then CTR ← CTR - 1
ctr_ok  ← BO_2 | ((CTR_{m:63} ≠ 0) ⊕ BO_3)
cond_ok ← BO_0 | (CR_{BI+32} ≡ BO_1)
if ctr_ok & cond_ok then
    if AA=1 then a ← ^{64}0 else a ← CIA
    NIA ← ^{m}0 || (a + EXTS(BD||0b00))_{m:63}
else            NIA ← ^{m}0 || (CIA + 4)_{m:63}
if LK=1 then LR ← CIA + 4
```

For us humans: very hard to write… and impossible to read!

Program flowcharts

A graphical, often informal representation of programs.



To produce machine language from a flowchart:
place instructions in memory; add branches; encode instructions.

Assembly language (1947)

A textual representation of machine language:

- mnemonics to name processor instructions;
- labels to name program points;
- comments to document the code.

Example: a counted loop that computes *n*!

```
;  Compute factorial of n.
;  n is passed in r1.
;  n! is left in r2.
;  r1 is clobbered.
        set r2, 1               ; initialize result to 1
again:  mul r2, r2, r1          ; multiply result by n
        sub r1, r1, 1           ; decrement n
        brnz r1, again          ; repeat until n = 0
                                ; here, result is n!
```

Macro-assemblers and “autocoders” (IBM, 1955)

Pseudo-instructions that represent often-used instruction sequences. Expanded by the assembler.

Example: a counted loop.

Source code

```
BEGIN_LOOP(lbl, r1, 0)
...
...
END_LOOP(lbl, r1, 100)
```

Code after expansion

```
      set r1, 0
lbl:
      ...
      ...
      add r1, r1, 1
      cmp r1, 100
      brlt lbl
```

The first control structures

The FORTRAN language (1957)

Complex expressions, using familiar algebraic notations, automatically translated to machine instructions (*FORmula TRANslator*).

FORTRAN source:

```
D = SQRT(B*B - 4*A*C)
X1 = (-B + D) / (2*A)
X2 = (-B - D) / (2*A)
```

Assembly code:

```
mul t1, b, b        sub x1, d, b
mul t2, a, c        div x1, x1, t3
mul t2, t2, 4       neg x2, b
sub t1, t1, t2      sub x2, x2, d
sqrt d, t1          div x2, x2, t3
mul t3, a, 2
```

But also: the first control structure!

Control in FORTRAN I

Counted loops:

```
       DO 100 I = 1, N
       ...
       ...
100:   ...
```

Repeatedly executes the lines between `DO` and `100` (excluded) with `I` taking successive values 1, 2, …, `N`.

Efficient compilation: test at end of loop, loop unrolling, etc.

Control in FORTRAN I

Labels and `GO TO`.

Three-way conditional branch `IF X L1, L2, L3`
(branch whether X < 0 or X $= 0$ or X > 0).

```
      IF X 701, 702, 702
701:  X = -X
702:  ...
```

Branching to a computed label.

```
      ASSIGN 100 TO DEST
      ...
      GO TO DEST
```

Fortran IV (1961): Boolean conditional branch.

```
      IF X .LT. Y GOTO 100
```

Structured commands in Algol (1960)

Expressions are formed from constants and variables. Likewise, Algol promotes the view that commands (statements, *s*) are constructed from elementary commands

- assignment: `x := expr`
- procedure call: `proc(arg1, arg2)`

combined using control structures

- sequence ("blocks"): `begin` s_1`;` s_2`; ... end`
- conditional: `if` *be* `then` s_1 `else` s_2
- counted loop: `for i :=` e_1 `step` e_2 `until` e_3 `do` *s*
- general loop: `while` *be* `do` *s*

A change of perspective on programs

At the syntax level: formal grammars such as "Backus-Naur form" (BNF); recursive algorithms for parsing.

$$
\begin{aligned}
\text{Commands:}\quad s ::= {} & x \ \texttt{:=}\ e \\
\mid\ & \mathit{proc}(e_1, \ldots, e_n) \\
\mid\ & \texttt{begin}\ s_1; \ldots; s_n\ \texttt{end} \\
\mid\ & \texttt{if}\ \mathit{be}\ \texttt{then}\ s_1\ [\texttt{else}\ s_2] \\
\mid\ & \texttt{while}\ \mathit{be}\ \texttt{do}\ s \\
\mid\ & \texttt{for}\ x\ \texttt{:=}\ e_1\ \texttt{step}\ e_2\ \texttt{until}\ e_3\ \texttt{do}\ s
\end{aligned}
$$

At the semantic level: (intuitive, then formal later on) a realization that the meaning of a command is entirely determined by that of its sub-commands.

Unstructured control in Algol

Algol also supports unstructured control, with jumps to labels that designate commands:

$$
\begin{array}{lll}
s ::= \ldots & & \\
\quad \mid L : s & & \text{command labeled } L \\
\quad \mid \texttt{go to } L \mid \texttt{goto } L \mid \texttt{go } L & & \text{jump to label } L
\end{array}
$$

A label has a scope, which is the block enclosing the definition of the label. We cannot jump into a block from outside the block.

Jump within ✔

```
begin
  integer i;
L:...
  goto L
end
```

Jump out ✔

```
begin
  integer i;
  ... goto L ...
end;
L: ...
```

Jump into ✘

```
goto L;
begin
  integer i;
L:...
end
```

Control structures + “goto”

This “Algol style” with structured control and `goto` quickly became the standard to describe and publish algorithms (since 1960 in *Comm. ACM*), instead of flowcharts.

ALGORITHM 95
GENERATION OF PARTITIONS IN PART-COUNT FORM

Frank Stockmal
System Development Corp., Santa Monica, Calif.

```
procedure partgen(c,N,K,G);  integer N,K;  integer array c;
  Boolean G;
comment This procedure operates on a given partition of the
  positive integer N into parts ≦ K, to produce a consequent
  partition if one exists. Each partition is represented by the
  integers c[1] thru c[K], where c[j] is the number of parts of the
  partition equal to the integer j. If entry is made with G = false,
  procedure ignores the input array c, sets G = true, and pro-
  duces the first partition of N ones. Upon each successive entry
  with G = true, a consequent partition is stored in c[1] thru c[K].
  For N = KX, the final partition is c[K] = X. For N = KX+r,
  1 ≦ r ≦ K−1, final partition is c[K] = X, c[r] = 1. When entry
  is made with array c = final partition, c is left unchanged and G
  is reset to false;
begin integer a,i,j;
        if ¬ G then go to first;
        j := 2;
        a := C[1];
test:   if a < j then go to B;
        c[j] := 1 + c[j];
        c[1] := a − j;
zero:   for i := 2 step 1 until j − 1
        do c[i] := 0;
        go to EXIT;
B:      if j = K then go to last;
        a := a + j × c[j];
        j := j + 1;
        go to test;
first:  G := true;
        c[1] := N;
        j := K + 1;
        go to zero;
last:   G := false;
EXIT: end partgen
```

Control structures + “goto”

We find this combination of “goto” and control structures in many imperative and object-oriented languages:
Algol 68, Algol W, Pascal, Ada, Simula, PL/I, C, C++, C#, Perl, Go, …

Languages with “goto” and counted loops, such as FORTRAN and BASIC, evolved by adopting Algol-style control structures (`if...else...` in FORTRAN 77, `do while` in FORTRAN 90).

Some imperative/object oriented languages without “goto”:
Modula-2 (1980), Eiffel (1986), Python (1991), Java (1995).

Other flavors of conditionals

Cascading Boolean tests:

`if` be_1 `then` s_1 `elsif` be_2 `then` s_2 `elsif ... else` s_n

also written (`cond` (be_1 s_1) (be_2 s_2) ... (`t` s_n)) in Lisp.

Case analysis on a value of integer type or enumerated type:

```
case (grade) of
  'A' : s1
  'B', 'C': s2
  'D' : s3
  'F' : s4
end
```

```
switch (grade) {
  case 'A': s1; break;
  case 'B':
  case 'C': s2; break;
  case 'D': s3; break;
  case 'F': s4; break;
}
```

Other flavors of loops

General loops:

- test at beginning: `while` *be* `do` *s*
- test at end: `do` *s* `while` *be* or `repeat` *s* `until` *be*
- test in the middle: `loop ...exit if` *be*`...end`

Counted loops with early exit conditions: (PL/I, Algol 68)

[`FOR` *index*] [`FROM` *first*] [`BY` *increment*] [`TO` *last*] [`WHILE` *condition*] `DO` *statements* `OD`

Loops that iterate over a collection of values:

`for` *item* `in` *collection*`:` *s*	(Python)
`for (`*Type item* `:` *collection*`) {` *s* `}`	(Java)

Early exits from loops

Exit from the enclosing loop:

`break` (C, C++, Java) / `exit` (Ada) / `last` (Perl)

Abort the current iteration and start the next one:

`continue` (C, C++, Java) / `next` (Perl)

Multi-level exit for nested loops:

- exit from the *N*-th enclosing loop — `break` *N* (Shell)
- exit from the loop labeled *L* — `break` *L* (Java)

```
xloop: for (int x = 0; x < dimx; x++)
  yloop: for (int y = 0; y < dimy; y++) {
    ... break xloop ... break yloop ...
}
```

Structured programming, structured control

The movement for structured programming (1965–1975)

A radical change of perspective on software.

Abandon the view of programs as

flowcharts and machine code

and start viewing programs as

a constructed, structured source text,
directly readable (without a flowchart on the side),
which we can reason about
(informally, then mathematically).

The manifesto for this movement: the book *Structured Programming* by Dahl, Dijkstra, and Hoare (1972).

The structured control controversy (1965–1975)

A dispute between two programming styles:

- Programs using many “goto”,
  obtained by naive transcription of a flowchart.
- Programs using mainly control structures (conditionals, loops), written directly, without a flowchart.

The slogan for this dispute:

Go to statement considered harmful

(Title of a short communication by Dijkstra in CACM 1968. The title was chosen not by the author but by the CACM editors.)

Early signs

Since the summer of 1960, I have been writing programs in outline form, using conventions of indentation to indicate the flow of control. I have never found it necessary to take exception to these conventions by using ***go*** *statements.*

I used the keep these outlines as original documentation of a program, instead of using flow charts ... Then I would code the program in assembly language from the outlines. Everyone liked these outlines better than the flow charts.

(Dewey Val Schorre, 1966)

Early signs

*If you look carefully you will find that surprisingly often a **go to** statement which looks back really is a concealed **for** statement. And you will be pleased to find how the clarity of the algorithm improves when you insert the **for** clause where it belongs.*

If the purpose [of a programming course] is to teach Algol programming, the use of flow diagrams will do more harm than good, in my opinion.

(Peter Naur, BIT, 1963).

Sparking the controversy

Go To Statement Considered Harmful

Key Words and Phrases: go to statement, jump instruction, branch instruction, conditional clause, alternative clause, repetitive clause, program intelligibility, program sequencing
CR Categories: 4.22, 5.23, 5.24

EDITOR:

For a number of years I have been familiar with the observation that the quality of programmers is a decreasing function of the density of **go to** statements in the programs they produce. More recently I discovered why the use of the **go to** statement has such disastrous effects, and I became convinced that the **go to** statement should be abolished from all "higher level" programming languages (i.e. everything except, perhaps, plain machine code). At that time I did not attach too much importance to this discovery; I now submit my considerations for publication because in very recent discussions in which the subject turned up, I have been urged to do so.

(E. W. Dijkstra, Communications of the ACM 11(3), 1968.)

A reflection on the difficulty to know "where we are" in a program execution, and which path was taken from the beginning.

Which “coordinates” to designate a point in the execution?

```
if (x == 0) {
   ...
} else if (y == 0) {
    ...          ⇐ YOU ARE HERE
} else {
    ...
    ...
}
```

Sequence and conditionals:
current program point + Boolean values of previous conditionals.

Which “coordinates” to designate a point in the execution?

```
while (x < y) {           ⇐ 2nd iteration
    while (a[x] != 0) {   ⇐ 5th iteration
        if (x == 0) {
            ...
        } else if (y == 0) {
            ...           ⇐ YOU ARE HERE
        } else {
            ...
            ...
        }
    }
}
```

Sequence, conditionals, and loops:

program point + Boolean values + iteration counts for all loops.

Which “coordinates” to designate a point in the execution?

```
while (x < y) {            ⇐ 2nd iteration
    while (a[x] != 0) {    ⇐ 5th iteration
        if (x == 0) {
            L:...
        } else if (y == 0) {
            ...            ⇐ YOU ARE HERE
        } else {
            ...; if (y < 10) goto L;
            ...
        }
    }
}
```

With unrestricted `goto`: *it becomes terribly hard to find a meaningful set of coordinates in which to describe the process progress* (Dijkstra).

Food for thought!

> *The tide of opinions first hit me personally in 1969, when I was teaching an introductory programming course for the first time. I remember feeling frustrated at not seeing how to write programs in the new style; I would run to Bob Floyd's office asking for help, and he usually showed me what to do.*
>
> (D. E. Knuth, 1974)

After Dijkstra's paper, a great many studies

- to see whether `goto`-free programming is really the best way to express program structure;
- to find out how to eliminate `goto`, on a case-by-case basis or systematically by program transformation.

Knuth's survey

D. E. Knuth, *Structured Programming with **go to** Statements*, Computing Surveys 6(4), 1974.

After a historical account of the controversy, Knuth studies, on well-chosen examples,

- the impact of `goto` elimination on code clarity (especially if Boolean flags must be added);
- the impact of `goto` elimination on performance;
- what control structures (beyond those of Algol) might help express program structure better.

Example: a hash table

Two arrays: `A[N]` containing the keys, `B[N]` containing the values.

```
void add(key k, data d)
{
    int i = hash(k);
    while (1) {
        if (A[i] == 0) goto notfound;
        if (A[i] == k) goto found;
        i = i + 1; if (i >= N) i = 0;
    }
  notfound: A[i] = k;
     found: B[i] = d;
}
```

Very concise code! But there are two `goto`...

Example: a hash table

Using a `while` loop:

```
void add(key k, data d)
{
    int i = hash(k);
    while (! (A[i] == 0 || A[i] == k)) {
        i = i + 1; if (i >= N) i = 0;
    }
    if (A[i] == 0) A[i] = k;
    B[i] = d;
}
```

The test `A[i] == 0` is duplicated.

Example: a hash table

Using a loop with early exits (`break`):

```
void add(key k, data d)
{
    int i = hash(k);
    while (1) {
        if (A[i] == 0) { A[i] = k; B[i] = d; break; }
        if (A[i] == k) {           B[i] = d; break; }
        i = i + 1; if (i >= N) i = 0;
    }
}
```

Minor duplication of the code `B[i] = d`, but no impact on performance.

Example: error handling with resource freeing

A system programming idiom in C.

```
        int retcode = -1;  // error
        int fd = open(filename, O_RDONLY);
        if (fd == -1) goto err1;
        char * buf = malloc(bufsiz);
        if (buf == NULL) goto err2;
        ...
        if (something goes wrong) goto err3;
        ...
        retcode = 0;   // success
err3: free(buf);
err2: close(fd);
err1: return retcode;
```

Avoids any duplication of the code that frees resources.

Example: error handling with resource freeing

A version without `goto` that suffers from code duplication:

```
int fd = open(filename, O_RDONLY);
if (fd == -1) return -1;
char * buf = malloc(bufsiz);
if (buf == NULL) { close(fd); return -1; }
...
if (something goes wrong) {
    free(buf); close(fd); return -1;
}
...
free(buf); close(fd); return 0;
```

Example: error handling with resource freeing

A version based on blocks `do ... while(0)` and on `break`.

Would be more robust with a multi-level `break`.

```
int retcode = -1;  // error
do { int fd = open(filename, O_RDONLY);
     if (fd == -1) break;
     do { char * buf = malloc(bufsiz);
          if (buf == NULL) break;
          do { ...
               if (something goes wrong) break;
               ...
               retcode = 0;   // success
          } while (0); free(buf);
     } while (0); close(fd);
} while(0); return retcode;
```

Expressiveness of structured control

The “goto” elimination problem

A technical question that plays a central role in the structured control controversy:

Is it always possible to transform an unstructured program (using `goto` and `if...goto`) into an equivalent structured program?

With or without code duplication?

With or without introducing additional variables?

A well-known result

Theorem

Any unstructured program (or, equivalently, any flowchart) is equivalent to a structured program comprising a single ***while*** *loop and one extra integer variable.*

A simple proof of the well-known result

```
1: s₁;
   if (b₁) goto 3;
2: s₂;
   goto 4;
3: s₃;

4: s₄;
   if (b₄) goto 1;
```

A simple proof of the well-known result

```
1: s₁;
   if (b₁) goto 3; else goto 2;
2: s₂;
   goto 4;
3: s₃;
   goto 4;
4: s₄;
   if (b₄) goto 1; else goto 0;
```

Reformat the program as basic blocks: sequences of assignments terminated by branches, either `goto` L or `if` be `then goto` L_1 `else goto` L_2.

A simple proof of the well-known result

```
1: s1;
   if (b1) goto 3; else goto 2;
2: s2;
   goto 4;
3: s3;
   goto 4;
4: s4;
   if (b4) goto 1; else goto 0;
```

Replace labels by numbers 0, 1, 2, …, with 0 being the label for the end of the program.

A simple proof of the well-known result

```
1: s1;
   if (b1) pc := 3; else pc := 2;
2: s2;
   pc := 4;
3: s3;
   pc := 4;
4: s4;
   if (b4) pc := 1; else pc := 0;
```

Replace each goto L by an assignment $\mathrm{pc} := L$
where pc is a new integer variable.

A simple proof of the well-known result

```
switch (pc) {
    case 1: s1;
            if (b1) pc := 3; else pc := 2; break;
    case 2: s2;
            pc := 4; break;
    case 3: s3;
            pc := 4; break;
    case 4: s4;
            if (b4) pc := 1; else pc := 0; break;
}
```

Turn each basic block into one case of a `switch` over the value of `pc`.

A simple proof of the well-known result

```
int pc = 1; while (pc != 0) {
   switch (pc) {
      case 1: s1;
              if (b1) pc := 3; else pc := 2; break;
      case 2: s2;
              pc := 4; break;
      case 3: s3;
              pc := 4; break;
      case 4: s4;
              if (b4) pc := 1; else pc := 0; break;
   }
}
```

Add a `while` loop to iterate the `switch` until `pc` is 0.

Iterating a transition function

If we view the original flowchart / unstructured program as a finite automaton, the previous proof amounts to constructing its transition function

$$\text{current state (in } \texttt{pc}) \longrightarrow \text{next state (in } \texttt{pc})$$

as a case analysis on the value of `pc`

```
switch (pc) { ... }
```

then to iterating this function until the final state is reached

```
while (pc != 0) { ... }
```

Variants:

one integer variable *pc* $\rightarrow$ several Boolean variables

one `switch` $\rightarrow$ a cascade of `if` ... `then` ... `else`.

The folk lore around this result

(David Harel, *On Folk Theorems*, CACM 23(7), 1980)

The result is often attributed to Böhm and Jacopini, *Flow diagrams, Turing machines, and languages with only two formation rules*, CACM 1966.

However, this paper shows a different result (resulting in several `while` loops) with more subtle techniques (local graph rewriting).

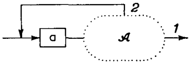


FIG. 13. Structure of a type I diagram

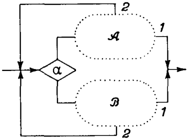


FIG. 14. Structure of a type II diagram

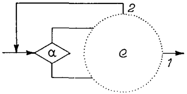


FIG. 15. Structure of a type III diagram

The folk lore around this result

The simple proof of the well-known result appears in 1967 in a letter by D. C. Cooper to the CACM editors.

It is mentioned later in tens of papers and books, often attributed to Böhm and Jacopini, or without attribution…

Harel traces the result back to Kleene (1936) !
(Every partial recursive function is the “minimization” of a primitive recursive function.)

Reductions between control structures

(S. Rao Kosaraju, *Analysis of Structured Programs*, JCSS 9, 1974.)

Let L_1 and L_2 be two languages that have the same base commands (assignments, function calls, ...) but differ on their control structures.

L_1 is reducible to L_2 if for each L_1 program, there exists an L_2 program that

- has the same base commands (no code duplication);
- uses no additional variables.

Example of reduction: `do-while` loop

The `do-while` loop (with test at end of iteration) is not reducible to the `while-do` loop (with test at beginning). Indeed, to translate

```
do s while be
```

we must either duplicate *s*

```
begin s; while be do s end
```

or introduce a Boolean variable

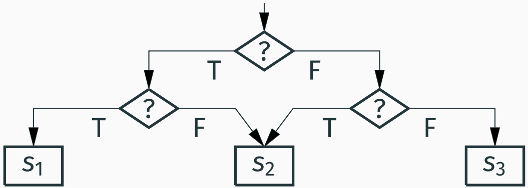
```
loop := true; while loop do begin s; loop := be end
```

In contrast, `do-while` is reducible to `while-do` + `break`:

```
while true do begin s; if not be then break end
```

Example of reduction: acyclic flowcharts

Cycle-free flowcharts are not reducible to `if-then-else`.



Either we duplicate s_2:

```
if ...
then if ... then s1 else s2
else if ... then s2 else s3
```

Example of reduction: acyclic flowcharts

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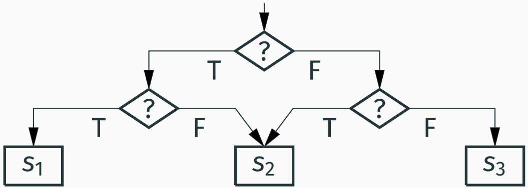

Or we add a Boolean variable:

```
do_s2 := false
if ...
then if ... then s₁ else do_s2 := true
else if ... then do_s2 := true else s₃;
if do_s2 then s₂
```

Example of reduction: acyclic flowcharts

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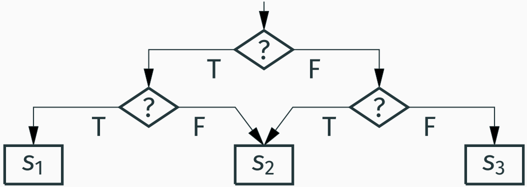


But we can reduce if we have loops with early exits (`break`):

```
loop
    if ...
    then if ... then begin s1; break end
    else if not ... then begin s3; break end;
    s2; break
endloop
```

Reducible control-flow graphs and structured control

(W. W. Peterson, T. Kasami, N. Tokura. *On the capabilities of while, repeat, and exit statements.* CACM, 16(8), 1973.)

The language of reducible control-flow graphs

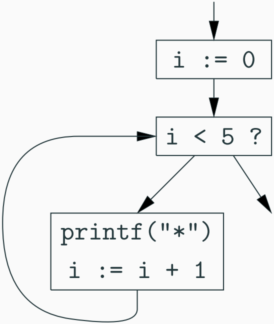
is Kosaraju-reducible

to a structured language with conditionals, infinite loops, and multi-level exits (`break` *N*, `continue` *N*).

Control-flow graphs (CFG)

A formalization of program flowcharts.

An intermediate representation used in many compilers.



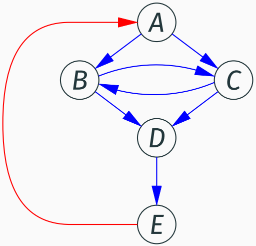
A CFG = a directed graph, with

- nodes = basic blocks
- edges = jumps

  (1 outgoing edge: `goto`;
  2 edges: `if-then-else`;
  N edges: `switch`)

Dominance

A node A dominates a node B if A is necessarily executed before B: any path from the root to B goes through A.



every node dominates itself

A dominates B, C, D, E

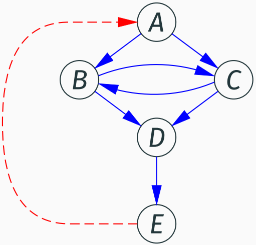
D dominates E

This leads to a classification of edges $A \to B$:

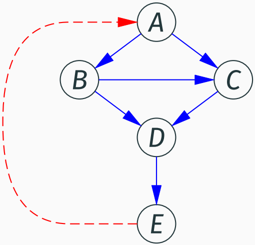
- it's a back edge if B dominates A;
- it's a forward edge otherwise.

Reducible control-flow graphs

A CFG is reducible if its forward edges form an acyclic graph.



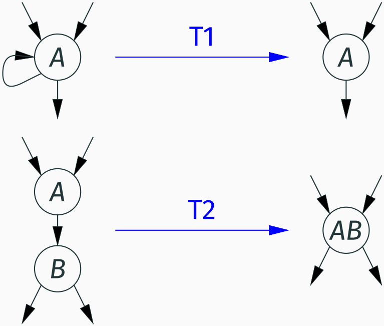
Irreducible



Reducible

Reducible control-flow graphs

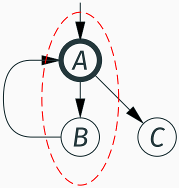
Many equivalent definitions, in particular:
a CFG is reducible if it can reduce to a single node by repeatedly applying transformations T1 and T2.



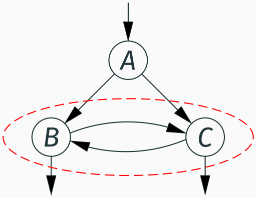
Loops in reducible CFGs

In a reducible CFG, each loop X (= strongly-connected set of nodes) is a natural loop:

- It has a single entry point $T \in X$, the loop head.
- T dominates every node in X.
- Every node in X has a path to T that stays within X.



Natural loop (head is A)



Non-natural loop

Control structures and reducibility

All the control structures viewed so far:

- conditionals: `if-then-else`, `switch`
- loops with test at beginning, at end, in the middle
- early exits: `return`, `break`, `continue`

produce reducible CFGs.

Control structures and reducibility

All the control structures viewed so far:

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Conversely: every reducible graph is the CFG of a program without `goto`, using only conditionals, infinite loops, and multi-level `break`/`continue` exits.

Constructing the structured program

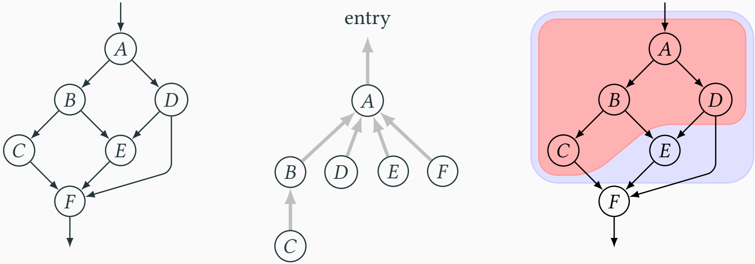
The historical algorithm by Peterson, Kasami, and Tokura (1973):

three passes; outlined in a proof.

A more recent algorithm: N. Ramsey, *Beyond Relooper: Recursive Translation of Unstructured Control Flow to Structured Control Flow*, ICFP 2022.

Recursion on the dominator tree for the graph.

Ramsey's algorithm



(a) Unusual control-flow graph (node E reachable two ways)

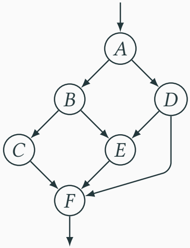
(b) Dominator tree

(c) Imposed nesting structure

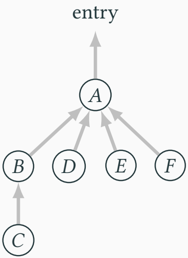
Each loop head produces an infinite `while true` loop.

The children in the domination tree that are also successors produce `if-then-else` conditionals. (A, B, C, D)

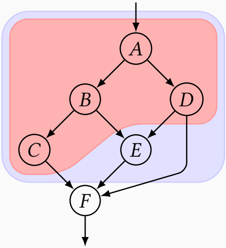
Ramsey's algorithm



(a) Unusual control-flow graph (node E reachable two ways)



(b) Dominator tree

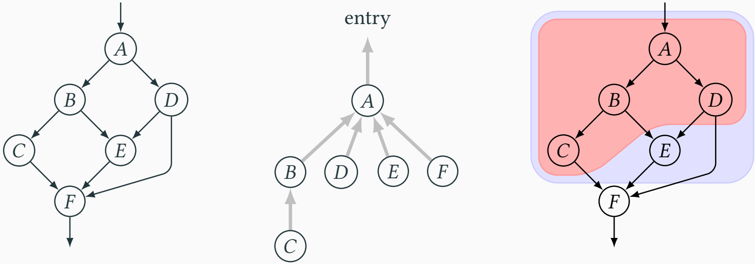


(c) Imposed nesting structure

The other children (E, F) are placed in blocks, nested according to reverse postorder.

Nontrivial edges become `break` N or `continue` N.

Ramsey's algorithm



(a) Unusual control-flow graph (node E reachable two ways)

(b) Dominator tree

(c) Imposed nesting structure

```
block {     // blue area
  block {   // red area
    if (A) { if (B) { C; break 2; } else { break 1; } }
      else { if (D) { break 1; } else { break 2; } }
  } E;
} F;
```

Summary

From goto to structured programming

From 1945 to 1975, programming practices and programming languages shifted

- from a "machine" view of control (jumps and labels), using program flowcharts to aid comprehension,
- to a structured view of control (conditionals, loops, etc.), making the source code the main representation of the program.

Next lecture: how to structure programs at a larger scale: subroutines, procedures, functions, generators, coroutines, …

References

References

A short, journalistic story of Algol:

- *ALGOL 60 at 60: The greatest computer language you've never used and granddaddy of the programming family tree*, The Register, 15/05/2020. https://www.theregister.com/2020/05/15/algol_60_at_60/

On structured programming:

- D. E. Knuth, *Structured Programming with **go to** Statements*, Computing Surveys 6(4), 1974.

On translating CFGs to structured programs:

- N. Ramsey, *Beyond Relooper: Recursive Translation of Unstructured Control Flow to Structured Control Flow (Functional Pearl)*, PACMPL 6, ICFP, 2022.