Will It Fit?

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Space Credits

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Space as a resource.

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Following Hofmann [1999], let $\Diamond 1$ represent one space credit.

- A space credit represents a permission to allocate one memory word.
- Space credits are non-negative numbers.
- Space credits can be split and joined: $\Diamond(n_1 + n_2) \equiv \Diamond n_1 * \Diamond n_2$

Warm Up: No GC

Without garbage collection, one can use the following two reasoning rules:

- "alloc" consumes space:

  $\{\ \Diamond n\ \}\ \text{alloc}\ n\ \{\ \lambda\ell.\ \ell \mapsto [0, \ldots, 0]\ \}$

- "free" produces space:

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and one can easily prove the following:

Adequacy theorem

If $\{\ \Diamond S\ \}\ t\ \{\ \lambda\_.\ \text{True}\ \}$ holds, then under heap limit S, the program t will not crash.

The GC Enters the Scene

- OCaml, Haskell, Scala, ... have garbage collection.
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- This makes programming easier...
- ...but makes reasoning more difficult. Where can space credits be recovered?

In this work:

- we answer this question,
- while supporting fine-grained shared-memory concurrency.

Free is a Ghost Update

Following Madiot and Pottier [2022], we make deallocation a logical operation:

$$\text{“}\ell\text{ is unreachable”} \;\ast\; \ell \mapsto [v_1, \ldots, v_n] \quad \Rrightarrow \quad \Diamond n$$

If a block is unreachable then by giving up its ownership one obtains space credits.

$\Diamond n$ means that n words are free or can be freed by the GC, once it runs.

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How does one prove that a location is unreachable?

Reachability

A location ℓ is reachable if:

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1. ℓ is not a root of any thread; and $\rightsquigarrow$ the pointed-by-thread assertion
2. ℓ has no (reachable) predecessor in the heap $\rightsquigarrow$ the pointed-by-heap assertion

Pointed-by-Heap Assertions

From Kassios and Kritikos [2013], Madiot and Pottier [2022], Moine et al. [2023].

- $\ell \hookleftarrow_1 A$ asserts that A is an over-approximation of the (reachable) predecessors of ℓ.

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- $\ell \leftarrow\!\!\dashv_1 \emptyset$ means that ℓ has no (reachable) predecessors.
- Pointed-by-heap assertions can be split and joined:

$$\ell \leftarrow\!\!\dashv_1 \{+\ell_1; +\ell_2\} \quad \equiv \quad \ell \leftarrow\!\!\dashv_{\frac{1}{2}} \{+\ell_1\} \ast \ell \leftarrow\!\!\dashv_{\frac{1}{2}} \{+\ell_2\}$$

- For greater comfort, we use signed multisets (not shown here).

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We introduce pointed-by-thread assertions:

- $\ell \Lleftarrow_1 \Pi$ asserts that Π is an over-approximation of the threads in which ℓ is a root.
- $\ell \Lleftarrow_1 \emptyset$ asserts that ℓ is not a root in any thread.
- Pointed-by-thread assertions can be split and joined:

$$\ell \Lleftarrow_{(p_1+p_2)} (\Pi_1 \cup \Pi_2) \quad \equiv \quad \ell \Lleftarrow_{p_1} \Pi_1 \ * \ \ell \Lleftarrow_{p_2} \Pi_2$$

Some Reasoning Rules: Alloc, Load and Fork

Allocation produces points-to, pointed-by-heap, and pointed-by-thread assertions:

$$\{ \Diamond n \}\ \pi\colon \mathsf{alloc}\ n\ \{ \lambda \ell.\ \ell \mapsto [0, \ldots, 0] \ * \ \ell \mathrel{\leftarrow\!\!\dashv}_1 \emptyset \ * \ \ell \mathrel{\Leftarrow\!\!\dashv}_1 \{\pi\} \}$$

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Reading a location ℓ' from the heap makes it a root in this thread:

$$\frac{0 \leq i < |\vec{v}| \qquad \vec{v}(i) = \ell'}{\{\, \ell \mapsto \vec{v} \ * \ \ell' \Lleftarrow_p \emptyset \,\}\ \pi \colon \ell[i]\ \{\, \lambda w.\ \ulcorner w = \ell' \urcorner \ * \ \ell \mapsto \vec{v} \ * \ \ell' \Lleftarrow_p \{\pi\} \,\}}$$

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Spawning a new thread transfers some roots to it. Here is a (simplified) rule:

$$\frac{\forall \pi'.\; \{\qquad\qquad\qquad \Phi \,\}\; \pi'\colon t \;\{\, \lambda().\; \mathsf{True} \,\}}{\{\qquad\qquad\qquad \Phi \,\}\; \pi\colon \mathsf{fork}\; t \;\{\, \lambda().\; \mathsf{True} \,\}}$$

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Some Reasoning Rules: Trimming and Logical Deallocation

Once ℓ is no longer a root in thread π, pointed-by-thread assertions can be trimmed:

$$\dfrac{\ell \notin \mathit{locs}(t) \qquad \{\, \ell \Lleftarrow_p \emptyset \;\; * \;\; \Phi \,\}\, \pi : t \,\{\, \Psi \,\}}{\{\, \ell \Lleftarrow_p \{\pi\} \;\; * \;\; \Phi \,\}\, \pi : t \,\{\, \Psi \,\}}\ \textsc{Trim}$$

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The logical deallocation rule can now be shown:

$$\ell \mapsto [v_1, \ldots, v_n] \;*\; \underbrace{\boxed{\ell \leftarrow_1 \emptyset}}_{\ell \text{ has no predecessors in the heap}} \;*\; \overbrace{\boxed{\ell \Lleftarrow_1 \emptyset}}^{\ell \text{ is not a root}} \;\Rrightarrow\; \ell \mapsto [v_1, \ldots, v_n] \;*\; \Diamond n \;*\; \underbrace{\dagger \ell}_{\text{deallocation witness}}$$

The points-to assertion is not consumed.

The Adequacy Theorem

A configuration is stuck if

- even after garbage collection has taken place,
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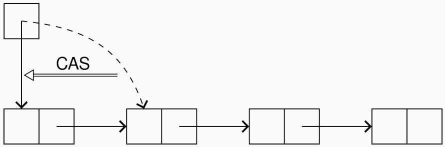
Adequacy Theorem

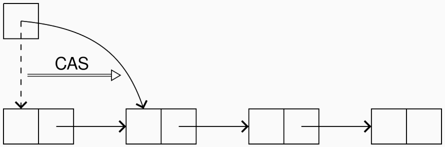
If $\{\,\Diamond S\,\}\,\pi\colon t\,\{\,\lambda\_.\ \mathrm{True}\,\}$ holds, then under heap limit S, t cannot become stuck.

The Case of Lock-Free Data Structures: Treiber's Stack

A linearizable lock-free stack, implemented as a reference on an immutable list.



push



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push pop

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  match xs with
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  | y::ys -> if CAS s xs ys then y else pop s
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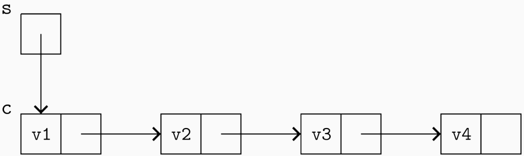
A Problematic Interleaving for pop



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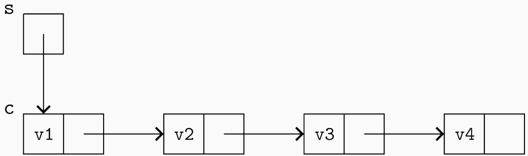


```
pop s
```

```
pop s;
pop s;
pop s;
...
```

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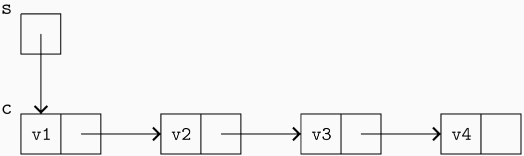


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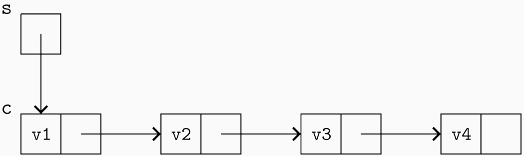


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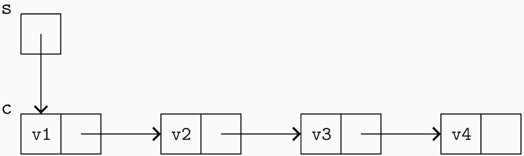


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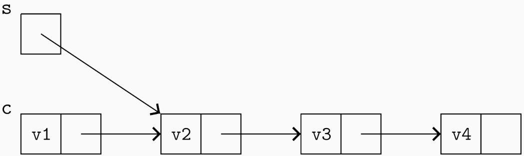


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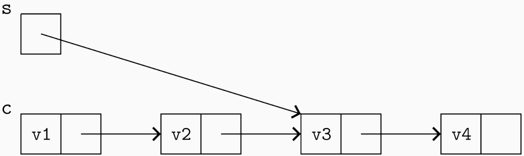


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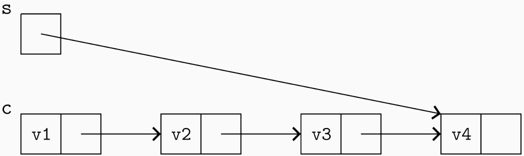


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- The sleeping thread holds the root c, therefore keeps the whole list reachable.
- We cannot prove that pop produces space credits!
- **Key idea**: c is a temporary root. Do not allow the GC to run while it is held.

LambdaFit

Our calculus, LambdaFit, has several novel features:

- protected sections, where the GC is disabled.
  - the GC can run only if all threads are outside protected sections
  - inside protected sections, allocation and divergence are forbidden
- polling points, which cannot be passed if any thread is waiting for the GC to run;
- there is a heap size limit S,
- and a memory allocation request blocks until it can be satisfied.

Temporary Roots

Inside a critical section, a root can be temporary – unobservable by the GC.

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Our reasoning rules take advantage of protected sections:

Key Idea

In the logical deallocation rule, ignore temporary roots.

Treiber's Stack, Fixed: pop

```
let rec pop s =
  enter ();
  let xs = !s in
  match xs with
  | nil -> assert false
  | y::ys ->
    if CAS s xs ys then (exit (); y) else (exit (); pop s)
```

- The variable `xs` is a root inside a protected section only.
- The GC can never observe this root.
- The problematic scenario disappears.

Inside and Outside Assertions

- Two new assertions: $\mathsf{outside}\,\pi$ and $\mathsf{inside}\,\pi\ T$.
- The parameter T is the set of temporary roots.

$$\{\,\mathsf{outside}\,\pi\,\}\ \pi : \mathsf{enter}\,()\ \{\,\lambda().\ \mathsf{inside}\,\pi\,\emptyset\,\} \qquad \{\,\mathsf{inside}\,\pi\,\emptyset\,\}\ \pi : \mathsf{exit}\,()\ \{\,\lambda().\ \mathsf{outside}\,\pi\,\}$$

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The assertion $\mathsf{inside}\,\pi\ T$ forms an escape hatch to the pointed-by-thread discipline.

$$\begin{array}{lcll}
\mathsf{inside}\,\pi\ T\ *\ \ell \Lleftarrow_p \{\pi\} & \Rrightarrow & \mathsf{inside}\,\pi\,(T \cup \{\ell\})\ *\ \ell \Lleftarrow_p \emptyset & \textsc{AddTemporary} \\
\mathsf{inside}\,\pi\ T\ *\ \ell \Lleftarrow_p \emptyset & \Rrightarrow & \mathsf{inside}\,\pi\,(T \setminus \{\ell\})\ *\ \ell \Lleftarrow_p \{\pi\} & \textsc{RemTemporary}
\end{array}$$

The Big Picture

$$\frac{\{\,\mathsf{inside}\,\pi\,(\,T \cap \mathit{locs}(t))\;\;*\;\;\Phi\,\}\,\pi\!: t\,\{\,\Psi\,\}}{\{\,\mathsf{inside}\,\pi\;T\;\;*\;\;\Phi\,\}\,\pi\!: t\,\{\,\Psi\,\}}\ \textsc{TrimInside}$$

The Big Picture

$$\frac{\{\,\mathsf{inside}\,\pi\,(T \cap \mathit{locs}(t)) \ \ast\ \Phi\,\}\ \pi : t\ \{\,\Psi\,\}}{\{\,\mathsf{inside}\,\pi\ T \ \ast\ \Phi\,\}\ \pi : t\ \{\,\Psi\,\}}\ \textsc{TrimInside}$$

- All of the previous rules remain sound, including logical deallocation.
- Therefore logical deallocation is not impeded by temporary roots.

Treiber's Stack, Fixed: push

```
let rec push s v =
  let xs = new_cell () in
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  enter ();
  let ys = !s in
  set_tail xs ys;
  if compare_and_swap s ys xs
  then exit ()
  else (exit (); push s v)
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Treiber's Stack, Fixed: push

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- That thread must logically deallocate ys,

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- and to do so, must also logically deallocate `xs`.

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- This is possible, as `ys` is a temporary root.

Treiber's Stack, Fixed: push

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```

- What if this thread falls asleep at the start of line 6 or 7?
- Some other thread may successfully pop `ys`.
- That thread must logically deallocate `ys`,
- and to do so, must also logically deallocate `xs`.
- This is possible, as `ys` is a temporary root.
- Once this thread wakes up, lines 6 or 7 access a logically deallocated block!
- This is possible, as logical deallocation does not consume the points-to assertion.

Specification of Treiber's Stack

$$\frac{\{\ \Diamond 2\ \}}{\dfrac{\langle\ \forall L.\ \mathsf{stack}\ s\ L\ \rangle \quad \pi : \mathsf{push}\ s\ v \quad \langle\ \mathsf{stack}\ s\ (v :: L)\ \rangle}{\{\ \lambda().\ \mathsf{True}\ \}}}$$

Specification of Treiber's Stack

Private precondition $\longrightarrow$

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Specification of Treiber's Stack

Private precondition

Public precondition

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Specification of Treiber's Stack

Private precondition

Public precondition

Public postcondition

$$\frac{\{\ \Diamond 2\ \}}{\dfrac{\langle\ \forall L.\ \mathsf{stack}\ s\ L\ \rangle \quad \pi : \mathsf{push}\ s\ v \quad \langle\ \mathsf{stack}\ s\ (v :: L)\ \rangle}{\{\ \lambda().\ \mathsf{True}\ \}}}$$

Specification of Treiber's Stack

Private precondition

Public precondition

Public postcondition

Private postcondition

$$\dfrac{\dfrac{\{\ \Diamond 2\ \}}{\langle\ \forall L.\ \mathsf{stack}\ s\ L\ \rangle}}{} \quad \pi : \mathsf{push}\ s\ v \quad \dfrac{\langle\ \mathsf{stack}\ s\ (v :: L)\ \rangle}{\{\ \lambda().\ \mathsf{True}\ \}}$$

Specification of Treiber's Stack

Private precondition → Public precondition ← Public postcondition ← Private postcondition →

$$\cfrac{\{\, \Diamond 2 \,\}}{\begin{array}{c}\langle\, \forall L.\ \mathsf{stack}\ s\ L \,\rangle \\ \pi : \mathsf{push}\ s\ v \\ \langle\, \mathsf{stack}\ s\ (v :: L) \,\rangle\end{array}}\over\{\, \lambda().\ \mathsf{True} \,\}$$

$$\cfrac{\{\, \mathsf{True} \,\}}{\begin{array}{c}\langle\, \forall v\ L.\ \mathsf{stack}\ s\ (v :: L) \,\rangle \\ \pi : \mathsf{pop}\ s \\ \langle\, \mathsf{stack}\ s\ L \,\rangle\end{array}}\over\{\, \lambda\, v'.\ \ulcorner v' = v \urcorner \ * \ \Diamond 2 \,\}$$

Polling Point Insertion

Polling points have no effect on reasoning.

$$\{\,\text{outside}\,\pi\,\}\;\pi\!:\text{poll}\,()\;\{\,\lambda().\;\text{outside}\,\pi\,\}$$

They can be automatically inserted by the compiler outside of protected sections.

If enough polling points are inserted,
then at all times,
every thread must reach a polling point in bounded time.

Safety and Liveness Theorems

Under the assumption that the program is verified:

$$\{\, \Diamond S \;\ast\; \text{outside}\,\pi \,\}\, \pi \colon t \,\{\, \lambda\_.\ \text{outside}\,\pi \,\}$$

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Then, at all times,

Safety

Every thread has terminated, is blocked on allocation, or can make progress.

Safety and Liveness Theorems

Under the assumption that the program is verified:

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Then, at all times,

Safety

Every thread has terminated, is blocked on allocation, or can make progress.

and assuming that enough polling points have been inserted, at all times,

Liveness

Every blocked thread is eventually unblocked.

Enough for Today!

Under review (journal paper):

Will it Fit? Verifying Heap Space Bounds of Concurrent Programs under Garbage Collection with Separation Logic

See also Alexandre's dissertation (upcoming).

Thank you for your attention!

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